#### **REVIEW**

of the official reviewer for the doctoral thesis of Zhami Bakytzhan on the topic "Investigation of hot rotating white dwarfs in general relativity", submitted for the degree of Doctor of Philosophy (PhD) in the specialty "6D060400 – Physics"

### 1. The relevance of the research topic and its connection with general scientific and national programs.

The work is devoted to theoretical studies of the properties of stable white dwarfs (WDs) based on the equation of state for the degenerate matter of cores of WDs, obtained by Chandrasekhar for zero core temperatures and with the involvement of subsequent modifications of this equation by various authors. The analysis is carried out taking into account the effects of finite temperatures of white dwarf cores, slow rotation, nuclear composition and general theory of relativity.

In the first chapter, the equations of state of the white dwarf substance are analyzed: the Chandrasekhar equation of state at zero temperature and at finite temperatures, the Salpeter equation and the Feynman-Metropolis-Teller equation of state at zero and at finite temperatures. In the second chapter, metrics of compact objects (Kerr, Hartle-Thorne, Fock, Sedrakyan-Chubaryan) are compared and their mutual correspondence is analyzed. In the third chapter, when considering the equilibrium configurations of white dwarfs, it is shown the accounting of the effects of general relativity, the reduction of equations to a dimensionless form and it is presented the results of numerical calculations for zero and finite temperatures. Calculations were also performed for the different nuclear compositions of white dwarfs: <sup>4</sup>He, <sup>12</sup>C, <sup>16</sup>O, <sup>20</sup>Ne, <sup>24</sup>Mg, <sup>28</sup>Si, <sup>56</sup>Fe. In the fourth chapter, the equilibrium configurations of rotating white dwarfs are considered in the framework of Newtonian gravity and general relativity and various parameters are analyzed, such as: mass, pressure, equatorial radius, moment of inertia, angular momentum, eccentricity, quadrupole moment, Love number with respect to central density and other relations of parameters. In the fifth chapter, when considering "I-Love-Q relations for white dwarf stars", it is used the general formalism for compact relativistic objects - Hartle formalism, which is widely used in the scientific community to describe relativistic objects such as neutron stars, quark stars and other exotic objects. The effects that cause rotation in the structure of WDs are investigated: the relations I-Q, I-e, Q-e, I-Love and Love-Q and it is shown that they are universal and independent of the equations of state for WDs and the chemical composition of the nucleus.

The relevance of the work is not in doubt. The investigations of white dwarfs are important for understanding the rate of expansion of the Universe, given their role as standard candles in type Ia supernova explosions. The justification of the contribution of white dwarfs to the chemical evolution of the galaxy before the transition of the star to this stage is a little doubtful, but this statement does not matter, since chemical evolution is not the subject of research.

The dissertation work has been fulfilled in accordance with the plans of the following fundamental scientific research works of the Science committee of the Ministry of Education and Science of the Republic of Kazakhstan on the themes: «Investigation of white dwarfs taking into account temperature and rotation in general relativity» (2015-2017); «Investigations on rotating and deformed objects in general relativity and relativistic astrophysics» (2013-2015).

#### 2. Scientific results and their validity.

Thesis by Zhami B.A. consists of five chapters, introduction and conclusion. The result of the work on the topic of the dissertation is a series of new and quite reliable scientific results.

The main scientific results of the work are:

- The mathematical and physical equivalence of the approximate stationary axially symmetric the Hartle-Thorne and Sedrakyan-Chubaryan solutions of Einstein field equations (up to second order in angular velocity) that describe the gravitational field of astrophysical compact objects including white dwarfs in the limiting case of slow rotation and small deformation.
- The effects of general relativity become significant in the mass-radius relation for white dwarf masses closer to the Chandrasekhar limit, thermal effects are most clearly pronounced for white dwarf masses lower than the Chandrasekhar limit, while rotation and nuclear composition are important for the entire mass range of white dwarfs.
- The core temperatures of white dwarf-satellites in the binary systems of millisecond pulsars PSR J1738+0333, PSR J1012+5307 and PSR J1911-5958A estimated to be in the range  $(1.55-6.5)\times10^7$  K,  $(1.4-2.4)\times10^7$  K and  $(0.4-9.5)\times10^7$  K, respectively.
- The I-Love-Q (the moment of inertia, the rotational Love number and the quadrupole moment) and I-Q-e (the moment of inertia, the quadrupole moment and the eccentricity) relations are universal and independent of the equations of state (Chandrasekhar and Salpeter equations of state) of white dwarfs.

## 3. The degree of validity and reliability of each scientific result (scientific provision), conclusions of the applicant, formulated in the thesis.

The validity and reliability of the research results are confirmed by publications in the journals indexed by Scopus and Web of Science with high impact factors and in the journals recommended by the Committee for the Control of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan. Approbation is confirmed by numerous reports at international conferences. The presentation of the text of the dissertation is perceived easily; it is written in a logical, clear language; it has a large volume, many sources and meets all the standards for dissertations.

## 4. The degree of novelty of each scientific result (scientific provision), conclusions of the applicant, formulated in the thesis.

The scientific novelty of the dissertation results of Zhami B.A. is not in doubt and consists in the following:

- A set of algebraic expressions relating the total mass, angular momentum and mass quadrupole moment of the Hartle-Thorne solution with the integration constants of the Sedrakyan-Chubaryan solution. Alternatively, the relevant multipole moments of both solutions have been calculated and it has been shown that they are identical. The mathematical and physical equivalence of the two metrics has also been proven.
- The mass-radius relations of white dwarfs taking into account the effects of general relativity, finite temperatures, nuclear composition and rotation were calculated.
- The core temperatures of white dwarf-satellites in the binary systems of millisecond pulsars PSR J1738+0333, PSR J1012+5307 and PSR J1911-5958A, have been estimated using the graphical method.
- The I-Love-Q and I-Love-e relations for white dwarfs have been shown to be universal and independent of the Chandrasekhar and Salpeter equations of state.

#### 5. The practical and theoretical significance of scientific results.

Firstly, the scientific results of the work are of certain value and novelty for the scientific community. Secondly, the results obtained in the work deepen the fundamental knowledge about the physics of white dwarfs and make a certain contribution to the development of the field of research. In addition, the results of the work can be directly used in further studies of problems of physics of white dwarfs and relativistic astrophysics.

### 6. Remarks, suggestions on the dissertation.

There are small typos, for example, in Figure 5.4, the dimension of the moment of inertia is indicated incorrectly:  $g \times cm$ , but  $g \times cm^2$  is necessary and in another figure 5.5,  $g \times cm^3$  is indicated. However, this does not spoil the overall positive impression of the work.

In conclusion, I want to express several wishes:

- 1) The use of a more complex approach in the study of different layers of the internal structure of white dwarfs, phase transitions, crystallization and other effects, which will allow us to come closer to explaining the parameters of the SDSS DR 10 and SDSS DR 12 catalogs.
  - 2) Apply the evolutionary approach to the study of white dwarfs.
- 3) It can be assumed the inclusion in the development of topics the consideration of radial and non-radial pulsations of white dwarfs, the analysis of which allows (as in ultrasound) to probe the internal structure of these stars, although this is a separate complex branch of astrophysics astroseismology.

# 7. Compliance of the dissertation with the requirements of «Rules for the award of scientific degrees».

The content of the dissertation on the topic "Investigation of hot rotating white dwarfs in general relativity" fully complies with the requirements of the rules for awarding the degree of Doctor of Philosophy (PhD) in the specialty "6D060400 – Physics" of the Committee for the Control of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan.

The applicant Zhami B.A. deserves to be awarded the degree of Doctor of Philosophy in the specialty "6D060400 – Physics".

Official reviewer, Candidate of Physical and Mathematical Sciences, Head of the Laboratory of Physics of Stars and Nebulae, V.G. Fesenkov Astrophysical Institute Shestakova L.I.

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